

A paradox in the global description of the multiverse

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ABSTRACT: We use an argument by Page to exhibit a paradox in the global description of the multiverse: the overwhelming majority of observers arise from quantum fluctuations and not by conventional evolution. Unless we are extremely atypical, this contradicts observation. The paradox does not arise in the local description of the multiverse, but similar arguments yield interesting constraints on the maximum lifetime of metastable vacua.

KEYWORDS: dS vacua in string theory, Superstring Vacua.

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1. Introduction

The landscape of string theory may explain the cosmological constant problem, but it has also given rise to new challenges. Of these perhaps the most formidable is our lack of techniques for making predictions in a theory with 10^{500} vacua. This task may require a drastic revision of our picture of the universe on the largest scales.

The standard, “global” picture is that of an eternally inflating “multiverse”, containing an infinite number of bubbles, or pocket universes, corresponding to each vacuum in the landscape. Each bubble is an infinite open universe, and if it has positive cosmological constant, it will itself harbor an infinite number of daughter universes.

The new, “local” picture is minimalist: It consists only of the spacetime region causally accessible from one worldline. This region has the shape of a causal diamond [1], defined as the overlap between the past light-cone of a late-time event with the future light-cone of an early event on the worldline. This is what one observer can probe, and *this is all there is*, as far as the semiclassical description of the universe goes.

As long as a theory can deal with *any* worldline and the associated causal diamond, it can describe all experiments that can be done in a semiclassical geometry. Thus, it is clearly sufficient to restrict to this region, and the economy of it may appeal to some. But is it necessary? What is wrong with the more intuitive, global point of view? What possible harm could it do?

In this paper, we first review known difficulties of the global point of view in section 2. While these difficulties are worrisome, they are not obviously fatal for the global description of cosmology. In section 3 we use an argument recently given by Page to exhibit a novel paradox: In the global description of the multiverse, almost all observers arise from random fluctuations, rather than by conventional evolution. This would make us extremely atypical and thus conflict with observation. In section 4 we discuss how the local viewpoint resolves

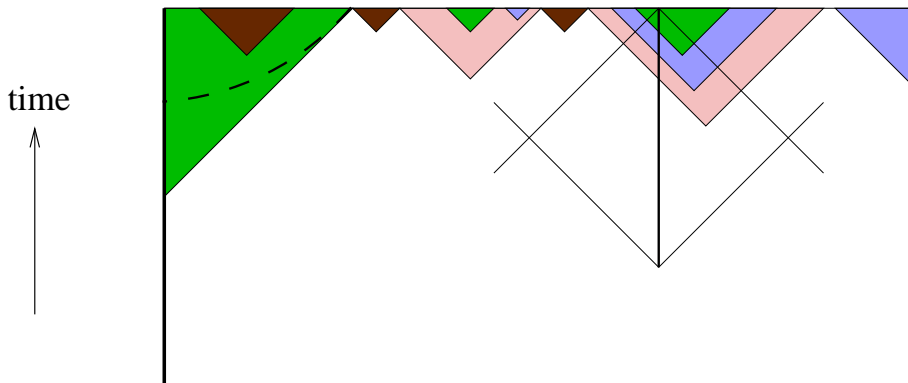


Figure 1: Globally, the universe contains regions corresponding to every vacuum in the landscape (shown in different colors). Each such region is an infinite open universe; the dashed line shows an example of an open equal-time hypersurface. The black diamond is an example of a spacetime region that is causally accessible to a single observer. Note that the global universe is not visible to any one observer.

the Page paradox. However, similar considerations yield a much weaker but nontrivial constraint on the lifetime of all vacua in the landscape that can contain observers.

The paradox discussed here should be regarded as analogous to the quantum xerox paradox in black hole physics. Just as in black hole physics, we believe the resolution lies in abandoning the global description.

2. Problems of the global picture

2.1 A predictivity crisis

We would like to predict low energy physics parameters observers are likely to observe. This requires statistical sampling of the theory landscape; an understanding of how the cosmological dynamics favors or disfavors the production of each vacuum; and finally, a sensible method for estimating the abundance of observers in each vacuum. (For example, parameters unique to a vacuum with no observers have zero probability of being observed.)

But in the global picture, it is not clear how to regulate the infinite number of infinitely large bubbles, so as to compare the number (or volume?) of regions corresponding to different vacua [2]. A number of proposals have been made, e.g. [3–5]. But we lack a convincing principle that would tell us which, if any, is correct. (Diffeomorphism invariance is sometimes cited, but any function of a such a measure will share the same property.)

In our view, this crisis of predictivity does not arise from a lack of sophistication or ingenuity in the prescriptions that have been proposed. It is the global point of view itself that is at fault. We are trying to regulate infinities that are figments of our imagination, and struggling to reign in volumes that cannot be seen by anyone without violating causality.

2.2 The FHW problem

In the global picture, it is natural to include volume expansion factors in the probability of vacua. All other things being equal, a vacuum that harbors a long period of slow-roll

inflation (say, a billion e -foldings) will be e^{10^9} times more likely than one with only 60 e -foldings.

Feldstein, Hall, and Watari [6] have pointed out that this leads to a problem, because in generic families of models of inflation, the duration of inflation either grows or shrinks monotonically with the size of the density perturbations produced. Thus $\delta\rho/\rho$ would be driven to 0 or 1 with exponential preference. Anthropic arguments [7] cannot help here — most observers would result from the exponential tails of Gaussian distributions and would find themselves alone in a hostile universe. But this is not what we see; the universe is full of galaxies like ours. Indeed, $\delta\rho/\rho \approx 10^{-5}$ is comfortably inside the anthropic window.

To be fair, this problem could be an artifact of an overly naive sampling of inflationary models. But the exponentially large volume factors make it difficult to come up with a credible sample of models such that $\delta\rho/\rho$ will be small but not exponentially small.

2.3 The quantum-xeroxing paradox for black holes

There is now considerable evidence that black hole formation and evaporation can be described as a unitary process by an outside observer. Yet, this would appear to create a paradox.

There is an instant of time at which both the collapsing star inside the black hole, and the Hawking radiation that allegedly carries away its quantum state, are in regions of negligible curvature, where semiclassical gravity should be valid. But this would mean that the quantum state of the star has been copied: it exists both inside and outside the black hole. This violates the linearity of quantum mechanics.

The paradox is resolved [8, 9] by noting that no observer can actually verify this violation. Either the observer stays outside, seeing only the Hawking radiation; or the observer falls in and sees only the star. There is no observer whose causal past can include both copies of the quantum state.

This suggests that the global viewpoint must be abandoned to avoid severe inconsistencies. It is not clear how such a conclusion can be confined to the context of black holes. Rather, one would expect it to apply generally, and thus in particular to cosmology.

However, no comparable, sharp paradox has been shown to plague the global description in the context of cosmology. We will now use a recent observation by Page to fill this gap.

3. The Page paradox

In a long-lived vacuum with positive cosmological constant, structure can form in two ways. Structure can form in the conventional way (through a period of inflation followed by reheating), or it can form spontaneously as a rare thermal fluctuation. Because de Sitter space is thermal, if the vacuum is sufficiently long-lived spontaneous structure formation will occur.

Observation indicates that the structure we see today was formed by conventional means. As explained by Dyson, Kleban, and Susskind [10], if structure forms spontaneously it is exponentially unlikely to be describable by a sensible semiclassical history.

For example, it is exponentially more likely for a single galaxy to fluctuate into existence than for the observed universe to form as a fluctuation. Among observers who form from thermal fluctuations, the vast majority will be close to the smallest fluctuation which can constitute an observer. This is the “Boltzmann’s Brain” paradox — within this framework, most observers are isolated brains which fluctuate from the vacuum in the absence of any other structure.

Page avoids the difficult problem of comparing different vacua in the multiverse and asks the following question: in the vacuum we find ourselves in, do more observers form conventionally or spontaneously? Since we seem to be in a spatially infinite universe, there are an infinite number of both types of observers. Page focuses on a finite comoving volume to regulate the spatial infinity. In a given comoving volume, a finite number of conventional observers form.¹ However, if our vacuum decays slowly enough that it eternally inflates, then the undecayed physical volume continues to grow with time. As a result, an infinite number of observers form spontaneously in a finite comoving volume. Page concludes that the decay rate of our vacuum must be fast enough that inflation is not eternal; otherwise, we would be infinitely atypical observers. The required decay time to avoid eternal inflation is of order the time scale set by the cosmological constant, $\Lambda^{-1/2}$, i.e., of order 10^{10} years.

It is not absurd to suggest that our vacuum will decay in a few billion years. However, as Page points out, his analysis suggests a stronger conclusion. If *any* vacuum which is capable of supporting observers eternally inflates, such a vacuum produces an infinite number of Boltzmann brains in a finite comoving volume. Presumably, this infinity would imply that a typical observer in the multiverse is a Boltzmann brain, and we would have to conclude that no vacuum capable of supporting observers eternally inflates. Since this stronger conclusion depends on comparing the relative probability of different vacua, other infinities could conceivably arise which would avoid this conclusion. We suspect, however, that any formulation of probabilities which relies on a global point of view will lead to the following conclusion: The observation that we observe conventional structure formation, together with the assumption that we are typical, implies that no vacuum capable of harboring observers eternally inflates.

Such a conclusion would be shocking, and is at odds with our current, admittedly crude, understanding of the string landscape. For example, in the toy model of ref. [11], our vacuum was estimated to have a lifetime of order $\exp(10^{10})$ if the number of fluxes is $O(100)$. With more fluxes, the upper exponent can be somewhat decreased but it would need to decrease to 3 for the lifetime to become of order ten billion years. This cannot be accomplished with a realistic number of fluxes if we still wish to solve the cosmological constant problem — a key motivation to consider the landscape in the first place.

In particular, we can show that the proposal of [4], which we consider the state of the art in globally inspired probability measures, suffers from the Page Paradox. In this proposal, the probability of measurements is computed in two steps. First, each vacuum is assigned an *a priori* probability P_i . The P_i encode the dynamics of eternal inflation, and

¹Some particularly rare fluctuations will happen to reproduce conventional evolution from a hot big bang, but they are exponentially less frequent than the production of isolated observers in an empty universe [10].

they are finite. In the second step, each vacuum is weighted by the number of observers within a unit comoving volume. The probability of observing vacuum i is proportional to the product of the a priori probability and the number of observers per comoving volume.

Page’s argument shows that if inflation is eternal, and the cosmological constant is small enough to fit one observer within the cosmological horizon, then the number of observers per comoving volume is infinite. So as long as at least one eternally inflating, observer-allowing vacuum exists with a finite a priori probability, the final probability distribution is zero for any vacuum which does not eternally inflate. Furthermore, in the eternally inflating vacua, observers are infinitely more likely to be Boltzmann brains than honest folk like ourselves.

One is left with a clear choice: either eternally inflating vacua admitting observers are shockingly absent from the string landscape, or the proposal [4] is incorrect. Though we cannot prove it, we expect the same difficulty to arise in any globally inspired proposal which has finite a priori probabilities for vacua with positive cosmological constant.

There is a deeper underlying reason for this problem. The global picture is, in a sense, an expansion about the least likely worldlines (those which fail to enter terminal vacua for an atypically long time). From the global viewpoint, the extreme unlikeliness of a worldline’s evolution is more than compensated by the exponentially large volume expansion factor it picks up. Hence, the global geometry of eternal inflation is dominated by regions which arose from the most unlikely evolution. Then it should not surprise us that the majority of observers can be similarly characterized, and arise from highly unlikely fluctuations.

4. A resolution: the local viewpoint

In the local viewpoint, the universe consists only of one (any) causally connected region of causal-diamond form. In our vacuum, for example, this region is the interior of the de Sitter horizon. (Strictly, it is overlap of the above “top cone” with the interior of a future lightcone which can be taken to start at reheating, but this restriction imposed by the bottom cone will not be needed here.)

In a vacuum with positive cosmological constant, the de Sitter horizon is finite, with area of order Λ^{-1} . Thus, the number of observers will be finite at any time. We can still be concerned about Boltzmann brains, and it remains true that we expect the first Boltzmann observers to show up after an exponentially long but finite time of order

$$t_{BB} \sim \exp(ER), \tag{4.1}$$

where E is the energy of the brain and $R = \Lambda^{-1/2}$ is the radius, and thus the inverse temperature, of the de Sitter space. (This time is the inverse of the Boltzmann factor, up to negligible prefactors. It can also be obtained directly from the entropy decrease of the heatbath, the cosmological horizon, when an object of energy E forms in de Sitter space [12].)

The difference is that in the global picture, your last chance to destroy the vacuum and prevent Boltzmann brains was much earlier, at a time of order R [13]. This is because the

universe is exponentially expanding after that time, and a bubble of new vacuum cannot catch up and completely replace the old vacuum.

In the local picture, the causal diamond is all there is. No-one can go and probe the exponentially large regions allegedly created by the cosmological expansion, so we do not consider them to be part of reality. What remains — the causal diamond — has constant asymptotic size R .² It can be easily replaced in its entirety by a new vacuum. Once a bubble forms, it will quickly expand out to the cosmological horizon.

Hence, all that we need is for the vacuum to decay before the first Boltzmann brains start appearing, after a time t_B .³ That is, we need its lifetime to be shorter than $\exp(ER)$. This is an exponentially weaker constraint than the lifetime of order R required in the global picture.

Nevertheless, the need to purge Boltzmann brains imposes interesting constraints on the landscape of string theory. It would seem to be important to avoid Boltzmann brains in all vacua. This includes de Sitter spaces with larger temperature, and thus higher brain creation rate. Because $T \sim 1/R$, the highest temperature is set by the smallest possible size of the Boltzmann brain: $R > R_B$. For example, with $E = 100$ kg and $R = 1$ m Boltzmann would be suppressed by $\exp(-ER) \sim \exp(-10^{45})$ (up to a negligible entropy factor). This relatively high rate applies not in our universe, but in some other vacuum with high enough cosmological constant to allow Boltzmann to fit snugly inside the cosmological horizon.

It seems plausible,⁴ but not obvious, that all vacua in the string landscape would decay sufficiently fast. The relevant comparison is the instanton action for the *fastest* decay channel of each vacuum, vs. the ER of a Boltzmann brain that could form in it. The decay would have to win this competition in all vacua. In other words, we need that the time to nucleate a Boltzmann brain is longer than the lifetime of the false vacuum,

$$t_{BB} > t_{\text{decay}}, \tag{4.2}$$

for all vacua in the landscape.

It is unclear how to characterize the requirements for an object to be a Boltzmann brain. One way of looking at it is that an ordered object must fluctuate out of the thermal bath of de Sitter space. That is, de Sitter space must fluctuate into a lower entropy configuration. The difference in entropy is a measure of how many computations the “observer” can perform before melting back into the heat bath [14]. For now, we do not try to quantify the necessary entropy difference, S_{BB} , but for an intelligent observer it must be quite large.

²Vacua with negative cosmological constant also have finite causal diamonds because they contain a big crunch. In the case of false vacua which can decay to $\Lambda = 0$, one might think an infinity appears even in the local analysis. Since such vacua must be supersymmetric, they cannot contain observers.

³This conclusion, as well as the discussion below, arose in e-mail discussions with T. Banks in July 2006.

⁴The basic idea is that the problem arises only in vacua with cosmological constant small compared to 1, or else not enough entropy will fit to admit an observer, Boltzmann or otherwise [1]. But to achieve a small cosmological constant requires combining a number of ingredients that can lead to accidental cancellation [11]. Hence there will be a large number of decay channels. It would be surprising if none of them satisfies eq. (4.2), especially since each ingredient should come from a fundamental theory and thus be tied to high energy scales.

The expected amount of time to create such an entropy difference scales as

$$t_{BB} \sim e^{S_{BB}} . \tag{4.3}$$

This time is very large. On the other hand, the typical decay time for a metastable vacuum is also exponentially large, since the decay is nonperturbative. As a result, (4.2) is a nontrivial constraint on the decay time, but a much weaker constraint than Page's.

For similar reasons (large inflationary expansion factors do not change the asymptotic size of the causal diamond), the local viewpoint also resolves the FHW problem (section 2.2). This is discussed in ref. [14], where it is also argued that unambiguous probabilities are obtained in this approach (section 2.1).

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